EE 508

Lecture 33

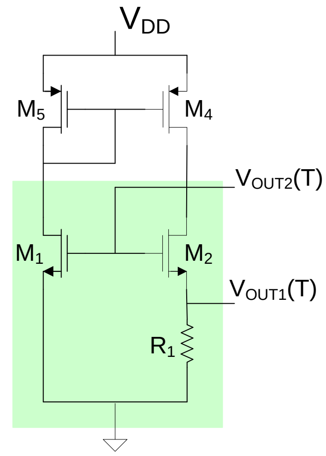
Oscillators, VCOs, and
Oscillator/VCO-Derived Filters

Review from last lecture:

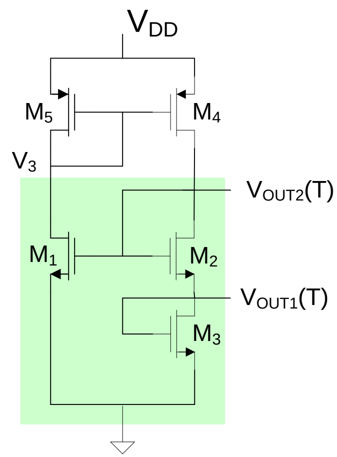
Only two of these circuits are useful directly as bias generators!

Inverse Widlar

Not stable equilibrium point !



(a)



(b)

Inverse Widlar

$$V_{01} = V_{Tn}\left(\frac{1-\sqrt{\frac{W_2L_1}{M_{IW}W_1L_2}}}{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-\sqrt{\frac{W_2L_1}{M_{IW}W_1L_2}}}\right)$$

$$V_{02} = V_{Tn}\left(\frac{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-2\sqrt{\frac{W_2L_1}{M_{IW}W_1L_2}}}{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-\sqrt{\frac{W_2L_1}{M_{IW}W_1L_2}}}\right)$$

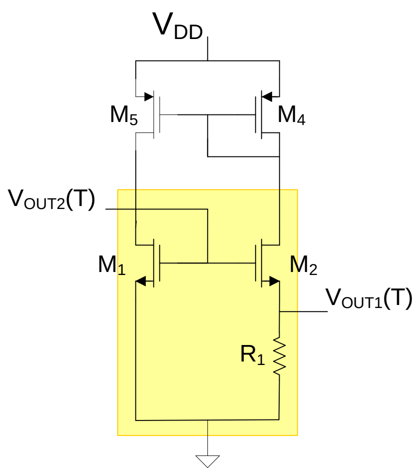
Widlar

$$V_{01} = \left(\frac{\theta_1}{2}\pm\sqrt{\frac{\theta_1 V_{Tn}}{2}+\left(\frac{\theta_1}{2}\right)^2}\right)\left(1-\sqrt{\frac{W_1L_2}{M_W W_2L_1}}\right)$$

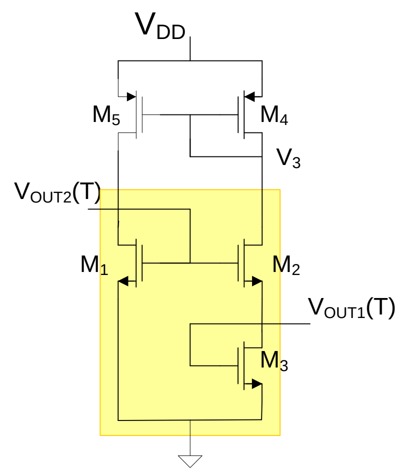
$$V_{02} = V_{Tn}+\frac{\theta_1}{2}\pm\sqrt{\frac{\theta_1 V_{Tn}}{2}+\left(\frac{\theta_1}{2}\right)^2}$$

$$I_{D1} = M_W I_{D2}$$

$$\theta_1 = \frac{M_W 2L_1}{R\mu_n C_{OX} W_1}$$



(c)



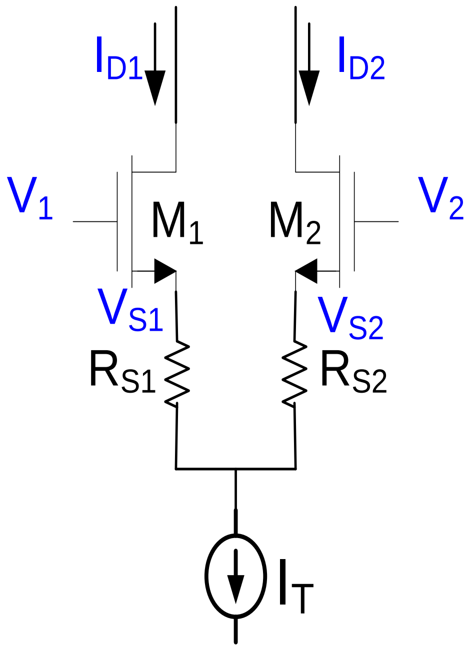
(d)

Widlar

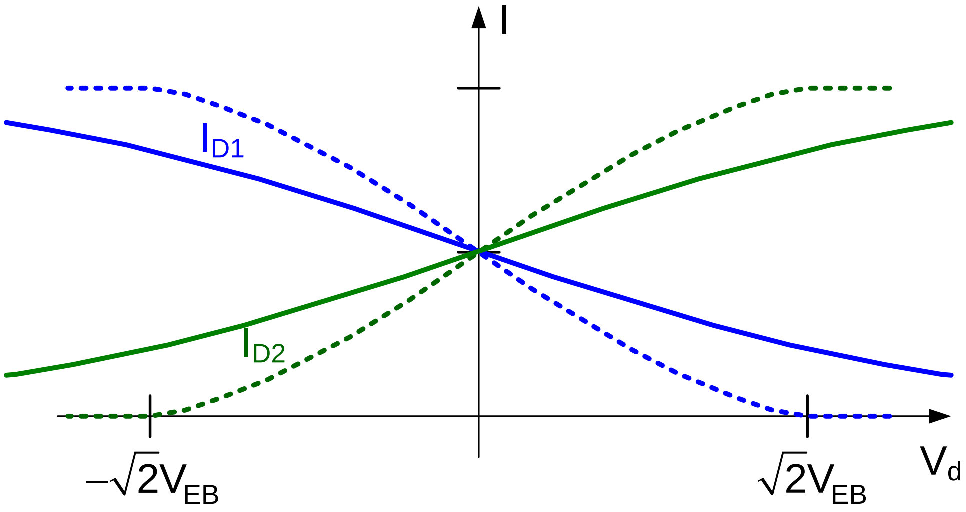
Not stable equilibrium point !

Review from last lecture:

Transconductance Linearization Strategies

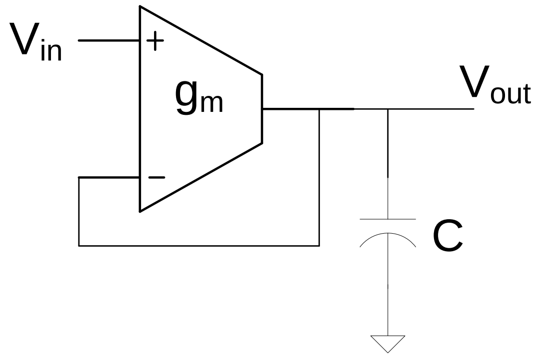


$$\sqrt{\frac{1}{\beta}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right) + R_S\left(I_T - 2I_{D1}\right) = V_d$$



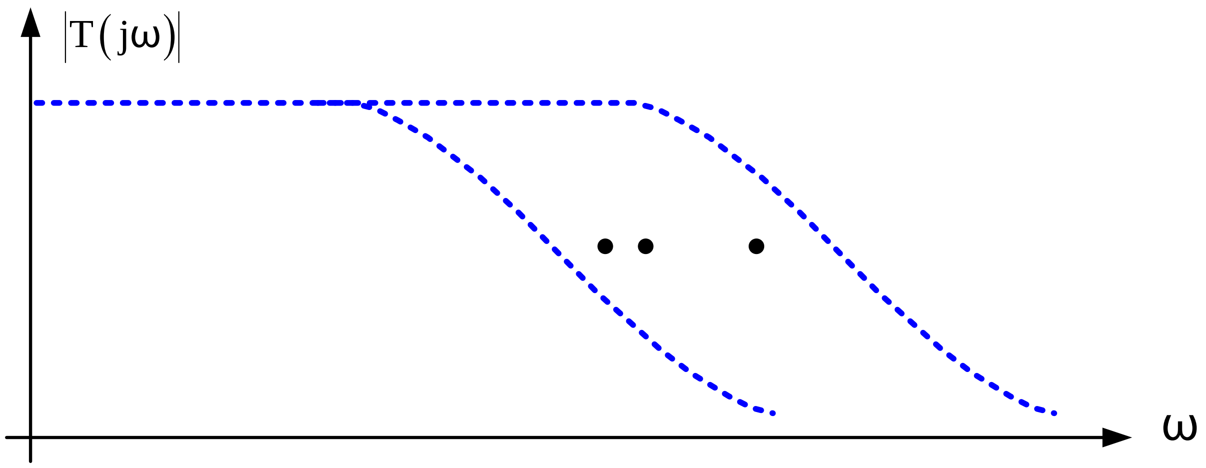
Programmable Filter Structures

It will be assumed that the transconductance gain can be programmed with either a dc current or a dc voltage



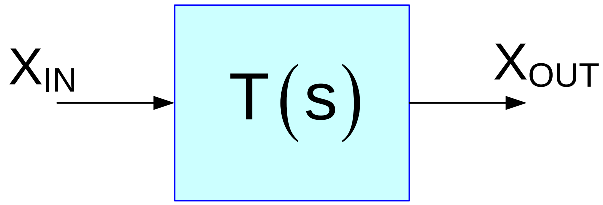
$$T(s)=\frac{g_m}{g_m+sC}$$

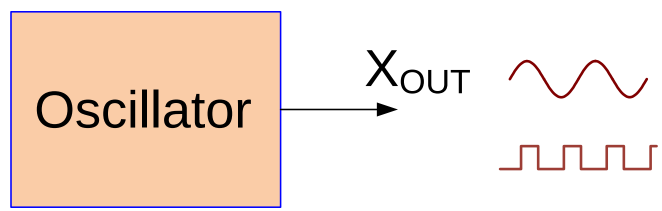
Programmable First-Order Low-Pass Filter



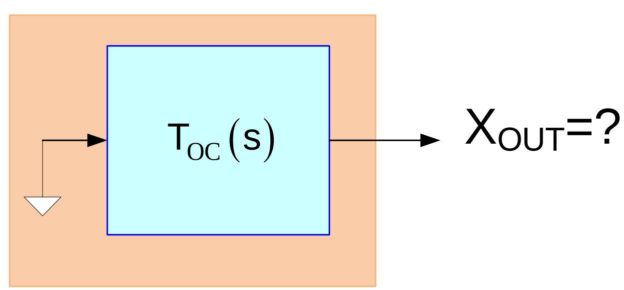
Question:

What is the relationship, if any, between a filter and an oscillator or VCO?

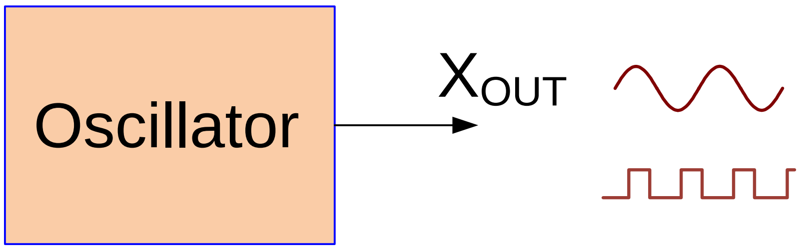


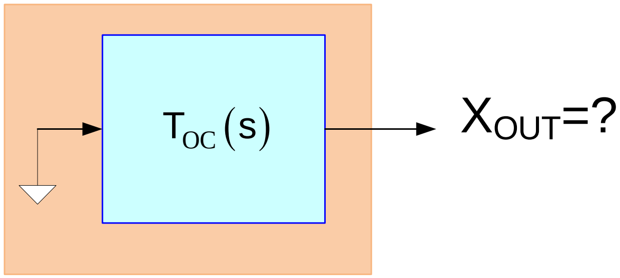


i.e. Can an oscillator be viewed as a filter with no input?



What is the relationship, if any, between a filter and an oscillator or VCO?



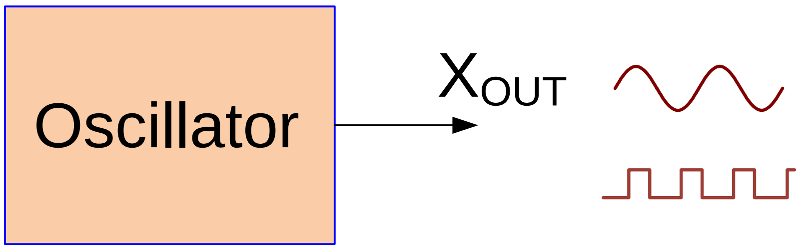


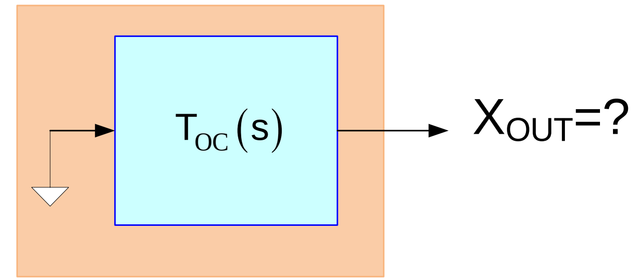
Will focus on modifying oscillator structures to form high frequency narrow-band filters

Claim: Narrow band filters are dependent primarily on the poles close to the imaginary axis and affected little by poles that are farther away

Goal: Obtain very high frequency filter structures

What is the relationship, if any, between a filter and an oscillator or VCO?



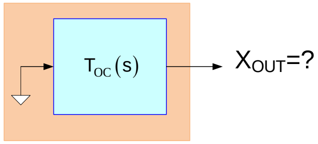


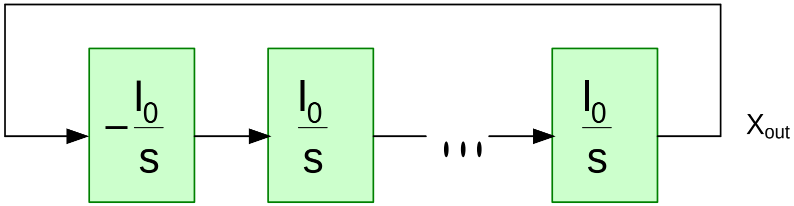
- When power is applied to an oscillator, it initially behaves as a small-signal linear network
- When operating linearly, the oscillator has poles (but no zeros)
- Poles are ideally on the imaginary axis or appear as cc pairs in the RHP
- There is a wealth of literature on the design of oscillators
- Oscillators often are designed to operate at very high frequencies
- If cc poles of a filter are moved to RHP is will become an oscillator
- Can oscillators be modified to become filters?

Oscillator Background:

Consider a cascaded integrator loop comprised of n integrators

This structure is often used to build oscillators





(assume an odd number of inverting integrators)

$$X_{OUT} = -\left(\frac{I_0}{s}\right)^n X_{OUT}$$

$$X_{OUT}\left(s^n + I_0^n\right) = 0$$

$$D(s) = s^n + I_0^n$$

Consider the poles of $D(s) = s^n + I_0^n$

$$s^n + I_0^n = 0$$

$$s^n = -I_0^n$$

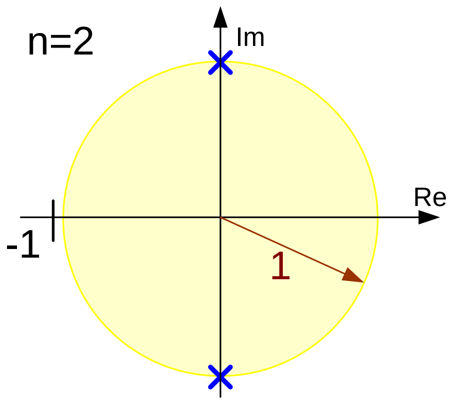
$$s = \left[-I_0^n\right]^{\frac{1}{n}}$$

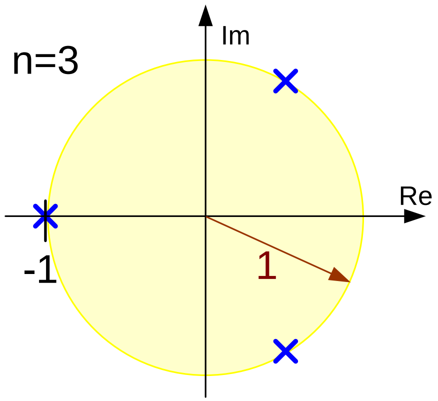
$$s = [-1]^{\frac{1}{n}} \left[I_0^n\right]^{\frac{1}{n}}$$

$$s = I_0 [-1]^{\frac{1}{n}}$$

Poles are the n roots of -1 scaled by I_0

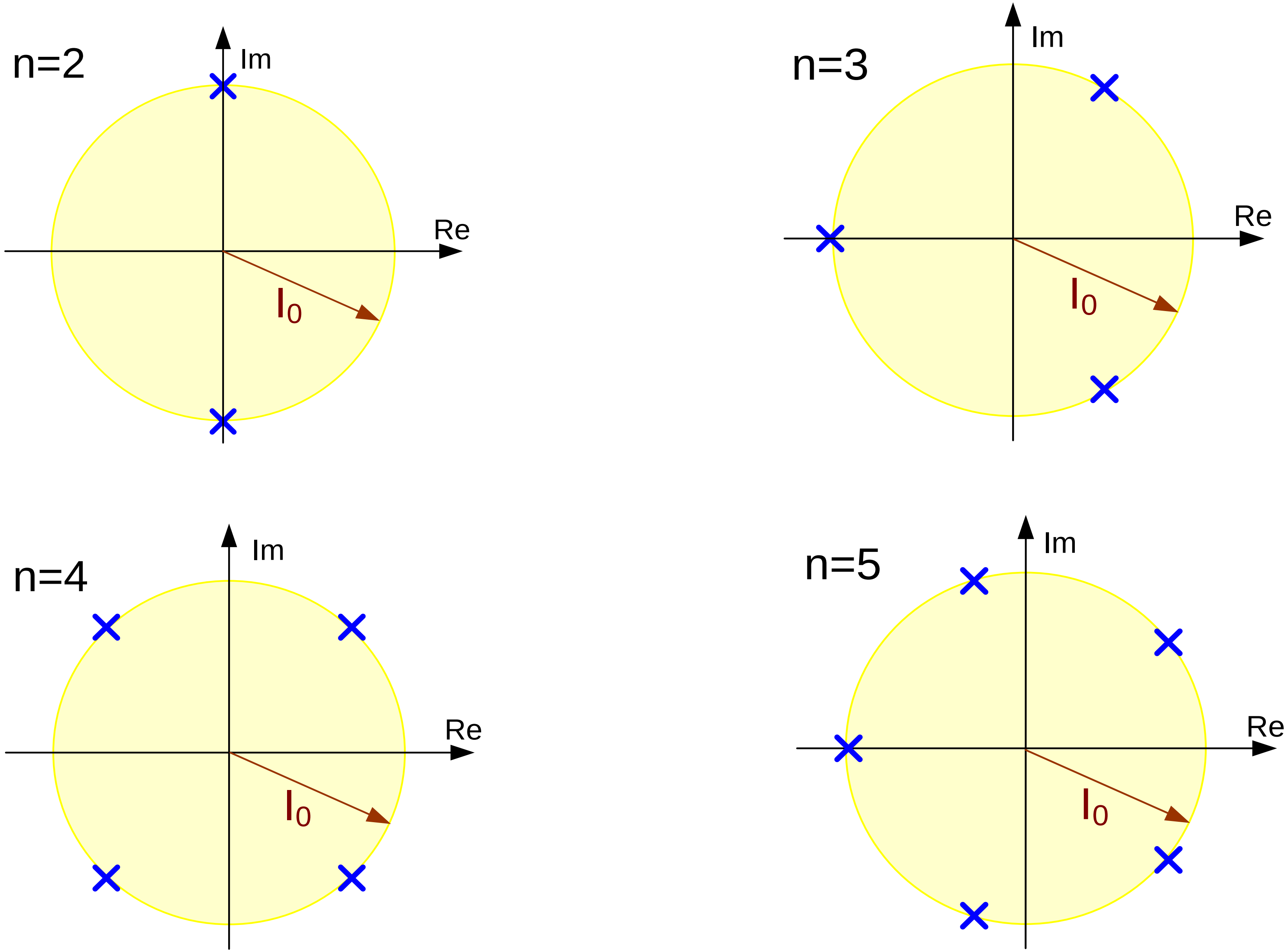
Roots of -1:

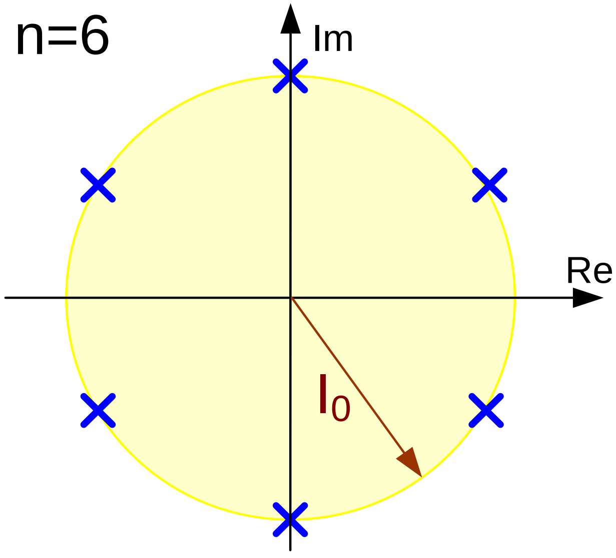


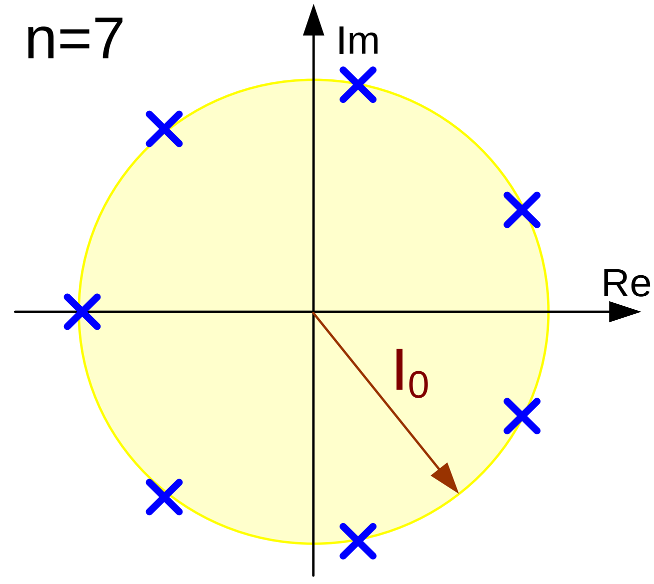


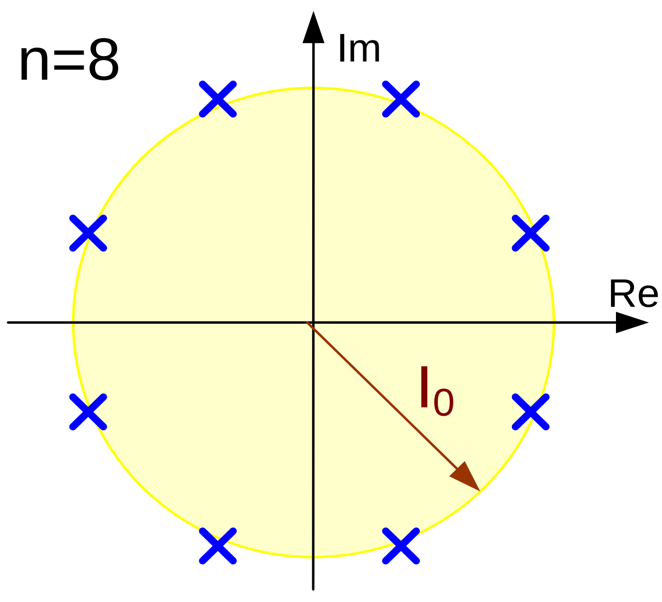
Roots are uniformly spaced on a unit circle

Consider the poles of $D(s) = s^n + I_0^n$




n=6
Im
Re
l_0


n=7
Im
Re
l_0


n=8
Im
Re
l_0

Some useful theorems

Theorem: A rational fraction $T(s) = \frac{N(s)}{\prod_{i=1}^{n}(s-p_i)}$ with simple poles can be expressed

in partial fraction form as $T(s)=\sum_{i=1}^{n}\frac{A_i}{s-p_i}$

where $A_i=(s-p_i)T(s)\big|_{s=p_i}$ for 1 ≤ j ≤ n

Theorem: The impulse response of a rational fraction T(s) with simple poles can be expressed as $r(t)=\sum_{i=1}^{n}A_i e^{p_i t}$ where the numbers A_i are the coefficients

in the partial fraction expansion of T(s)

Theorem: If p_i is a simple complex pole of the rational fraction T(s), then the partial fraction expansion terms in the impulse response corresponding to p_i and p_i^* can be expressed as

$$\frac{A_i}{s-p_i}+\frac{A_i^*}{s-p_i^*}$$

Theorem: If $p_i = \alpha_i + j\beta_i$ is a simple pole with non-zero imaginary part of the rational fraction T(s), then the impulse response terms corresponding to the poles p_i and p_i^* in the partial fraction expansion can be expressed as

$$|A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

where θ_i is the angle of the complex quantity A_i

Observe r(t) term corresponding to any complex pole pair is real !

Theorem: If all poles of an n-th order rational fraction T(s) are simple and have a non-zero Imaginary part, then the impulse response can be expressed as

$$\sum_{i=1}^{n/2} |A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

where θ_i ,$A_{i,}\alpha_i$, and β_i are as defined before

Theorem: If an odd-order rational fraction has one pole on the negative real axis at α_0 and n simple poles that have a non-zero Imaginary part, then the impulse response can be expressed as

$$A_0 e^{\alpha_0 t} + \sum_{i=1}^{n/2} |A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

where θ_i ,$A_{i,}\alpha_i$, and β_i are as defined before

Observe r(t) is real for both even and odd n !

Consider the following 3-pole situation

Poles of $D(s) = s^n + I_0^n$

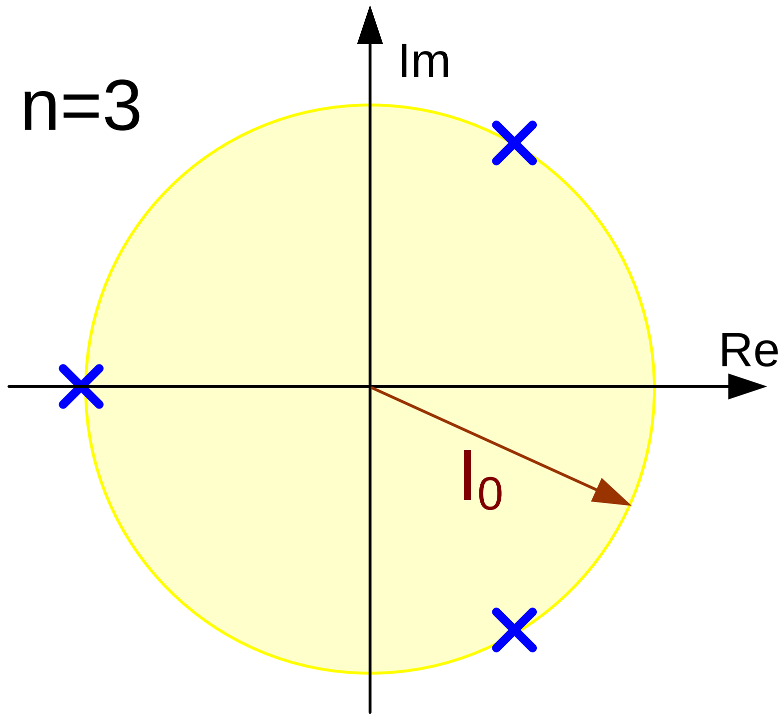
n=3

$\theta = 60^o$

$\alpha = I_0 \cos(60^o)$

$\beta = I_0 \sin(60^o)$

0.5	-0.866025404
0.5	0.866025404
-1	3.67545E-16



for cc pole pair:

$\alpha = 0.5\ I_0$

$\beta = 0.866\ I_0$

Oscillatory output at startup with any small impulse input:

$$|A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

Starts at $\omega = 0.866 I_0$ and will slow down as nonlinearities limit amplitude

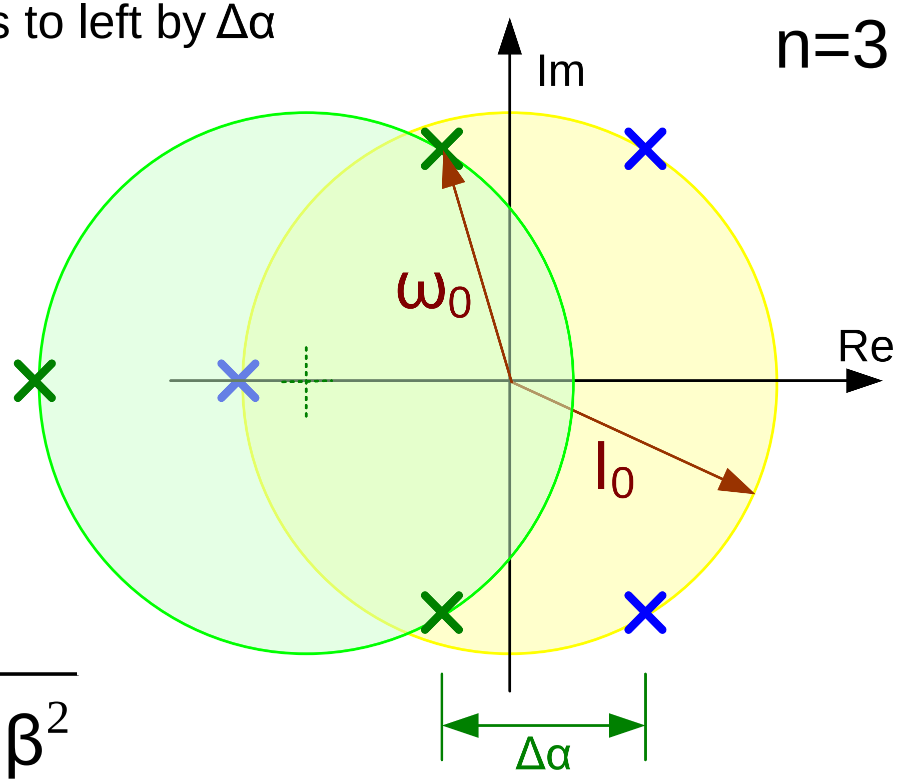
Consider the following 3-pole situation

Poles of $D(s) = s^n + I_0^n$

Consider moving all poles to left by $\Delta\alpha$

$\beta = 0.866\ I_0$

$\alpha = 0.5\ I_0 - \Delta\alpha$



$$\omega_0 = \sqrt{(\alpha - \Delta\alpha)^2 + \beta^2}$$

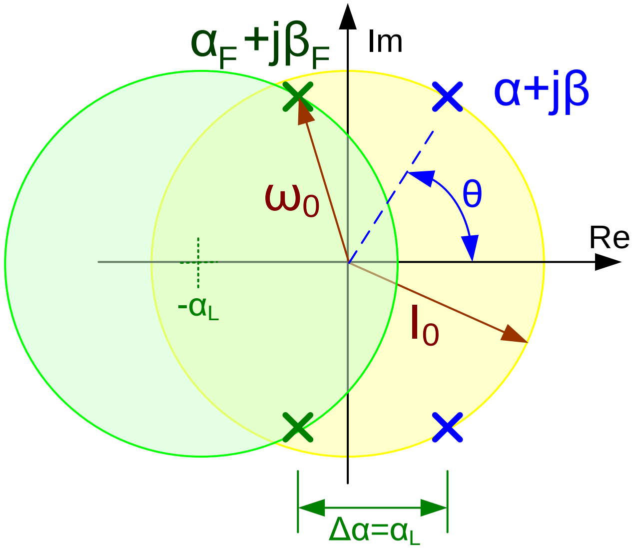
So, to get a high ω_0, want β as large as possible

Consider now the filter by adding a loss of α_L to the integrator

Will now determine α_L and I_0 needed to get a desired pole Q and ω_0 by moving all poles so that right-most pole pair is dominant high-frequency pole pair of filter

The values of α and β are dependent upon I_0 but the angle θ is only dependent upon the number of integrators in the oscillator or VCO

$$\alpha + j\beta = I_0 \left(\cos\theta + j\sin\theta \right)$$



Define the location of the filter pole to be

$$\alpha_F + j\beta_F$$

It follows that

$$\beta_F = \beta \qquad \alpha_F = \alpha - \alpha_L$$

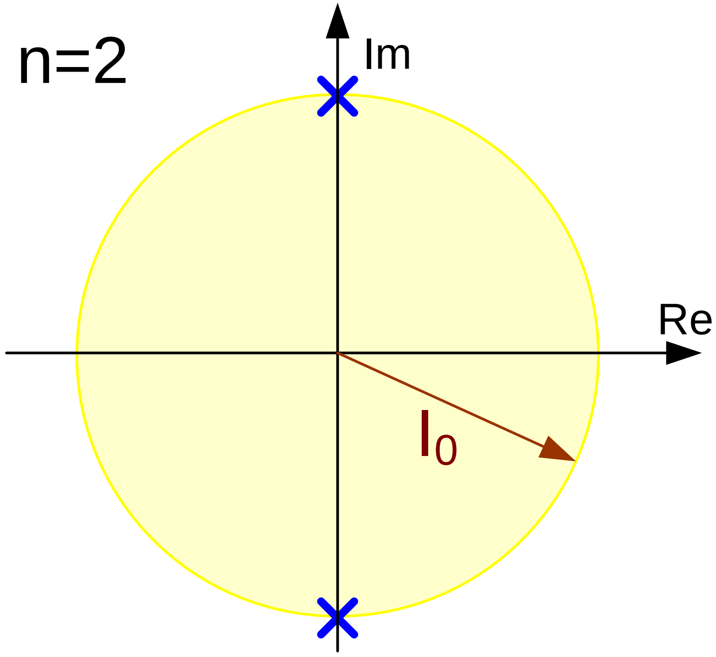
The relationship between the filter parameters is well known

$$\beta_F = \frac{\omega_0}{2Q}\sqrt{4Q^2-1} \qquad \alpha_F = -\frac{\omega_0}{2Q}$$

Thus for any n

$$I_0 = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \qquad \alpha_L = \frac{\omega_0}{2Q} + I_0\cos\theta = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1}$$

Will a two-stage structure give the highest frequency of operation?

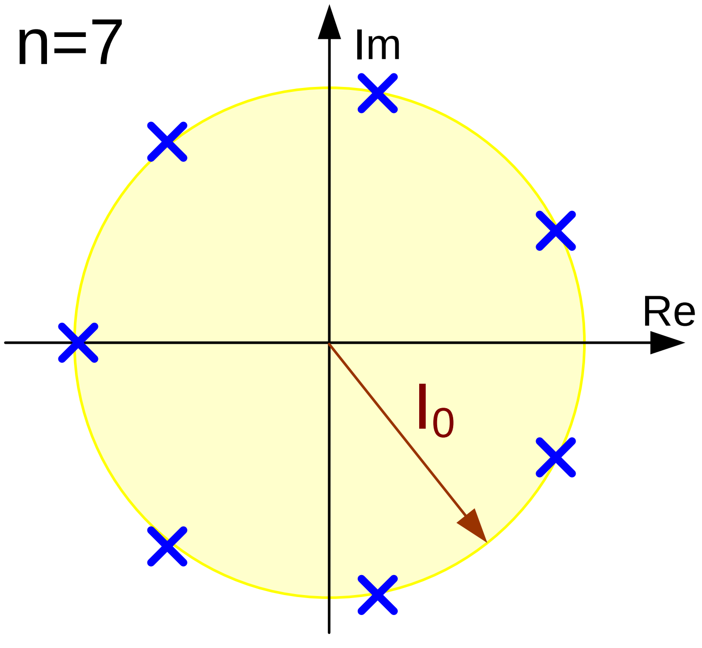


$$\omega_0 = \sqrt{(\alpha - \Delta\alpha)^2 + \beta^2} \longrightarrow \omega_0 = \sqrt{(-\Delta\alpha)^2 + \beta^2}$$

- Even though the two-stage structure may not oscillate, can work as a filter!
- Need odd number of inversions in integrators
- Can add phase lead if necessary

Oscillator Background:

What will happen with a circuit that has two pole-pairs in the RHP?

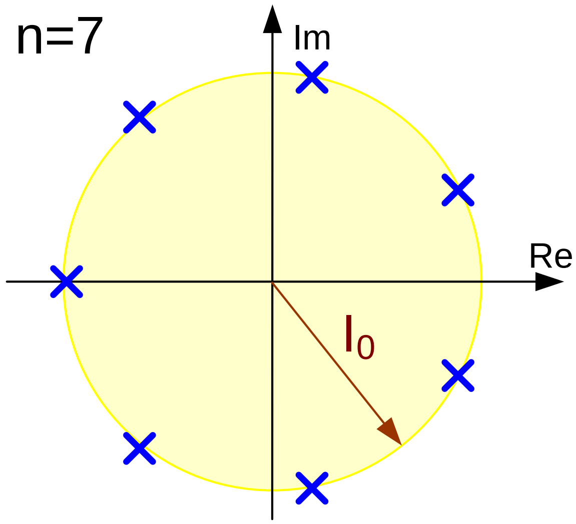


General form of response for odd number of poles:

$$A_0 e^{\alpha_0 t} + \sum_{i=1}^{n/2} |A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

The impulse response will have two decaying exponential terms and two growing exponential terms

What will happen with a circuit that has two pole-pairs in the RHP?



-0.62349	-0.781831482
0.222521	-0.974927912
0.900969	-0.433883739
0.900969	0.433883739
0.222521	0.974927912
-0.62349	0.781831482
-1	3.67545E-16

$\alpha_1=0.2225$ $\beta_1=0.974$

$\alpha_2=0.9009$ $\beta_2=0.4338$

Consider the growing exponential terms and normalize to $I_0=1$

$$|A_1|e^{\alpha_1 t}\cos(\beta_1 t+\theta_1) + |A_2|e^{\alpha_2 t}\cos(\beta_2 t+\theta_2)$$

At t=145 (after only 10 periods of the lower frequency signal)

$$r = \left.\frac{e^{\alpha_2 t}}{e^{\alpha_1 t}}\right|_{t=145} = \frac{e^{.9009\bullet 145}}{e^{.2225\bullet 145}} = 5.2\text{x}10^{42}$$

The lower frequency oscillation will completely dominate !

What will happen with a circuit that has two pole-pairs in the RHP?

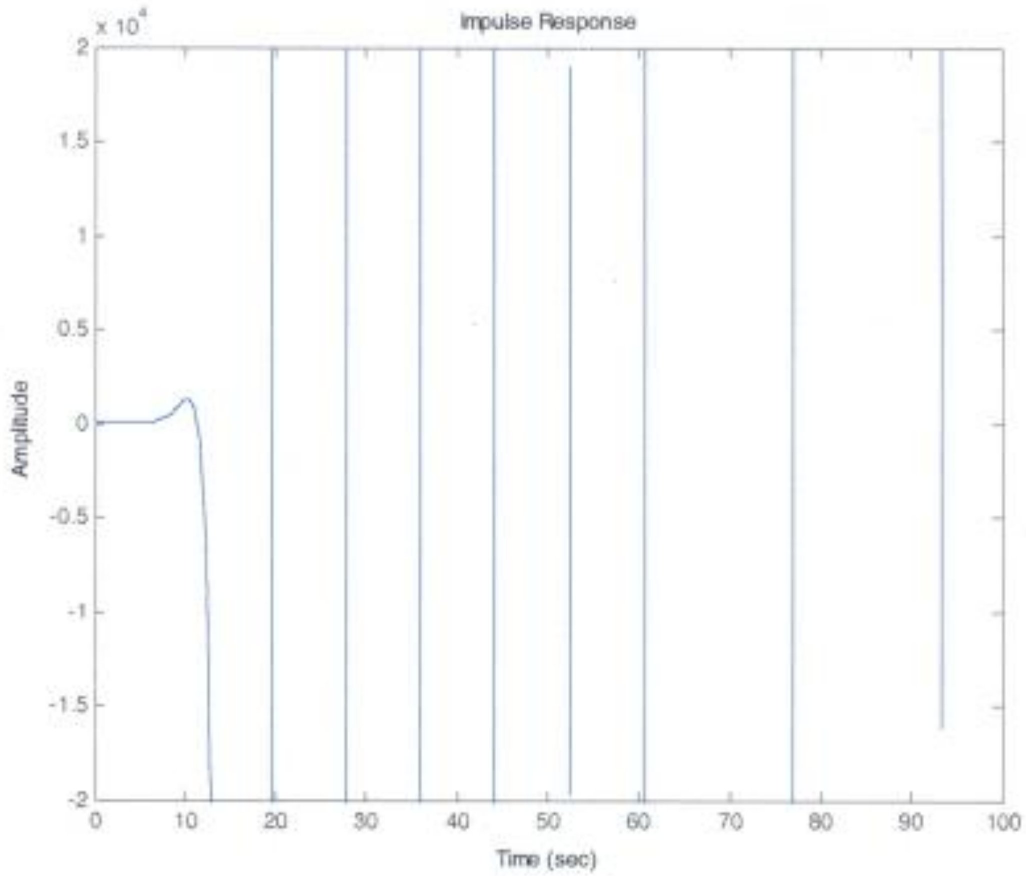
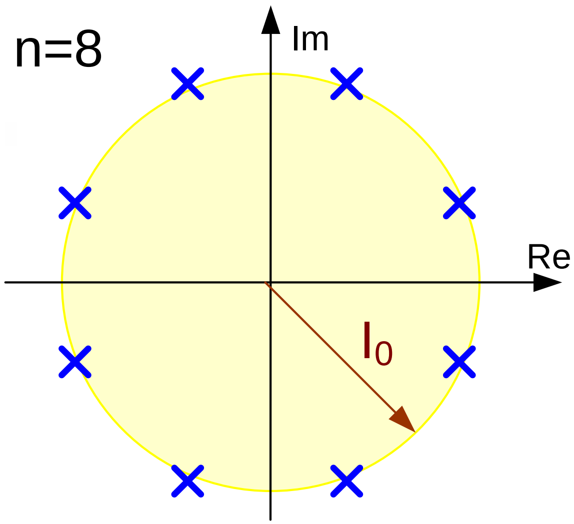


Figure 14 N=8 impulse response

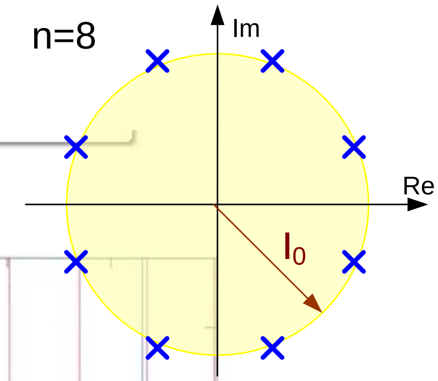


Thanks to Chen for these plots

Can only see the lower frequency component !

What will happen with a circuit that has two pole-pairs in the RHP?

Thanks to Chen for these plots



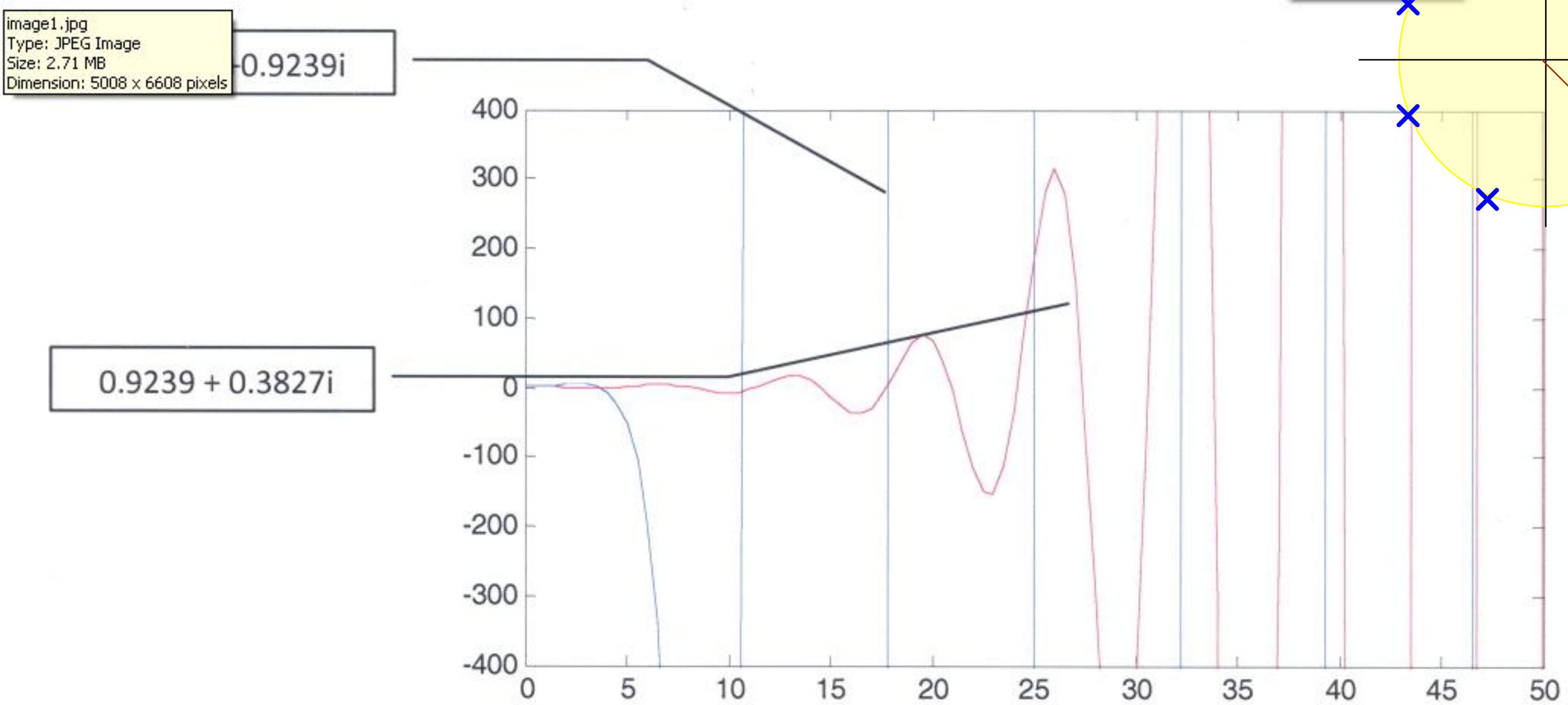
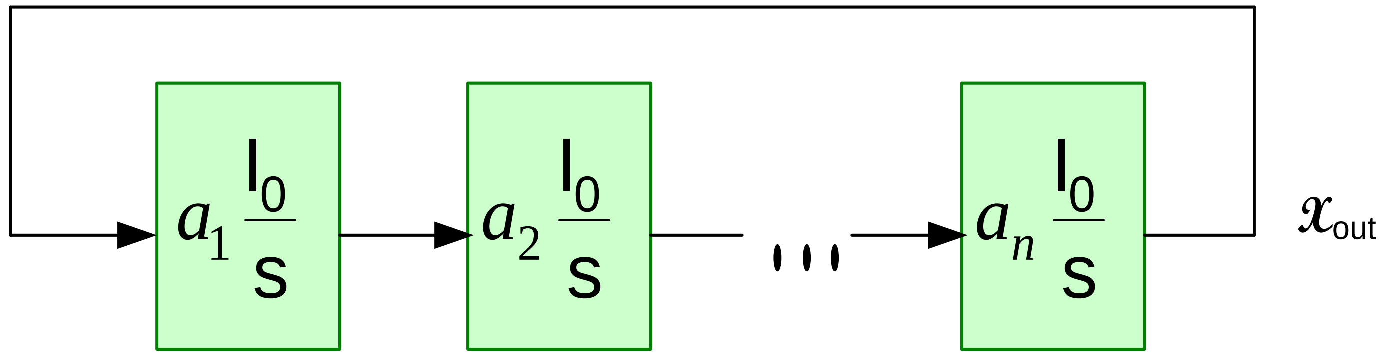


Figure 7 N=8 the impulse response of two poles

After even only two periods of the lower frequency waveform, it completely dominates !

How do we guarantee that we have a net coefficient of +1 in D(s)?

$$D(s) = s^n + I_0^n$$



$$x_{out} = \left(\prod_{i=1}^{n} a_i \left(\frac{I_0}{s} \right) \right) x_{out}$$

$$a_i \in \{-1,1\}$$

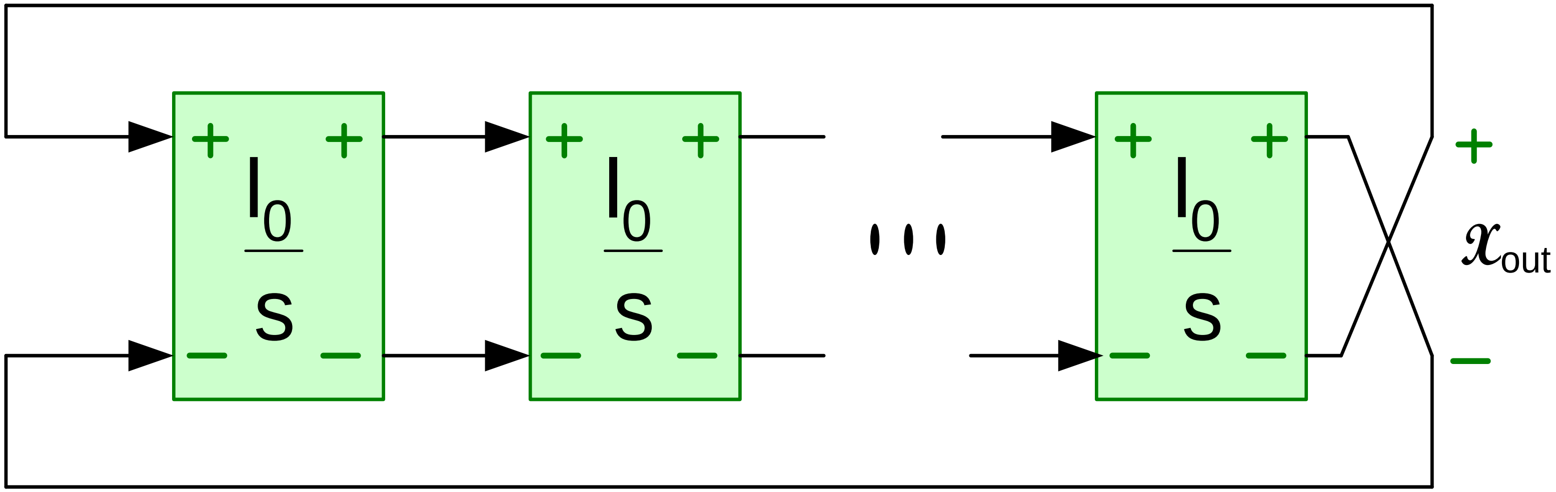
$$D(s) = s^n - \left(\prod_{i=1}^{n} a_i \right) I_0^n \longrightarrow \prod_{i=1}^{n} a_i = -1$$

Must have an odd number of inversions in the loop !

If n is odd, all stages can be inverting and identical !

How do we guarantee that we have a net coefficient of +1 in D(s)?

$$D(s) = s^n + I_0^n$$



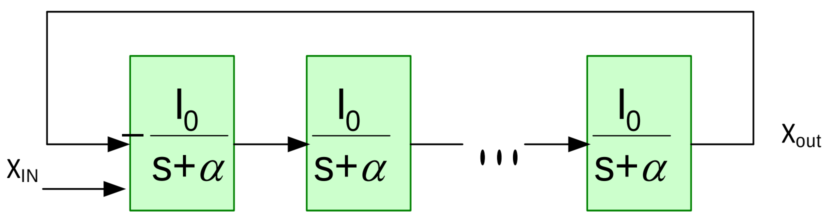
If fully differential or fully balanced, must have an odd number of crossings of outputs

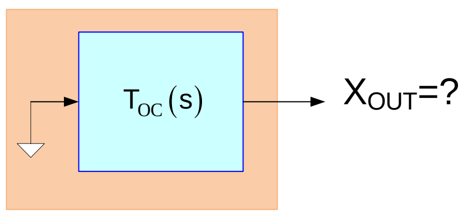
Applicable for both even and odd order loops

Inputs to Oscillator-Derived Filters:

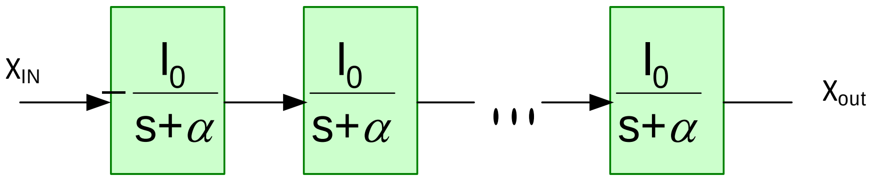
Most applicable to designing 2nd-order high frequency narrow band filters

- Add loss to delay stages
- Multiple Input Locations Often Possible
- Natural Input is Input to delay stage

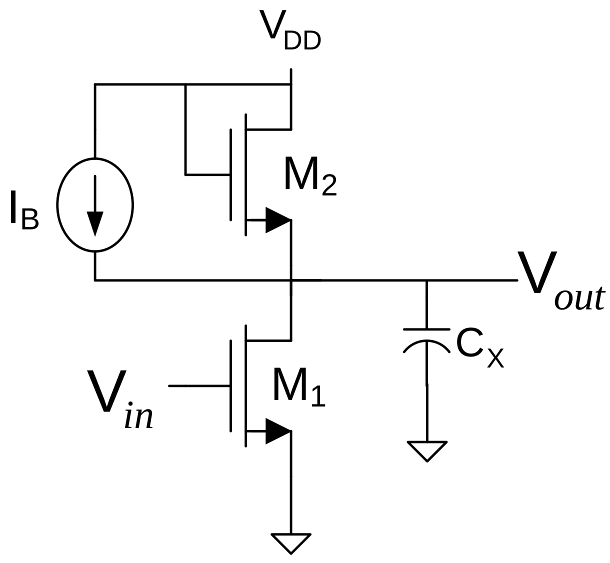




- Add loss to delay stages
- Often Just Salvage Stages (drop feedback loop)
- Natural input is input to delay stage



A lossy integrator stage

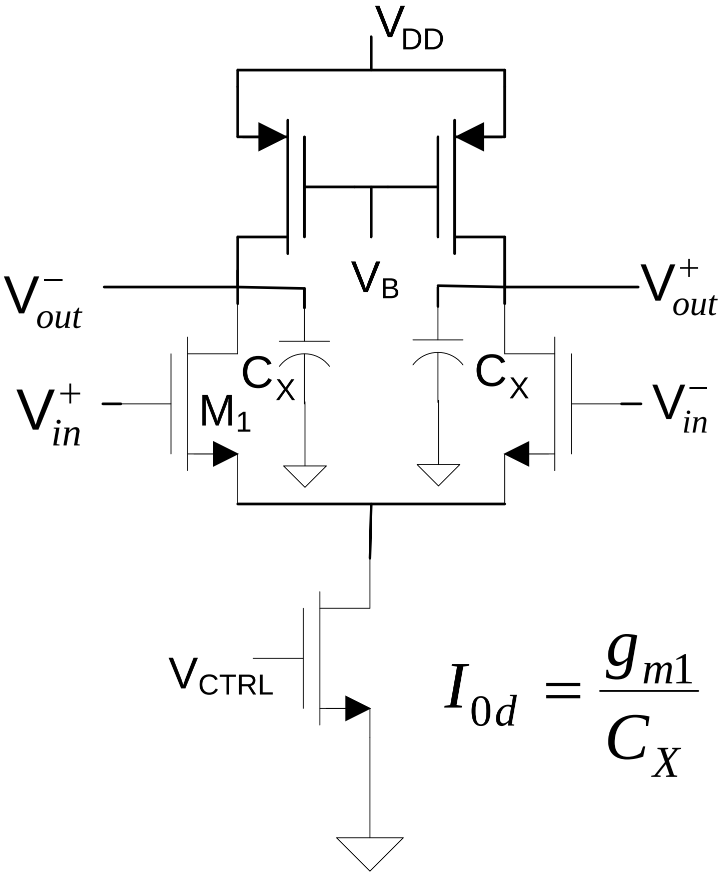


$$I(s) = \frac{-g_{m1}/C_X}{s+g_{m2}/C_X}$$

$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

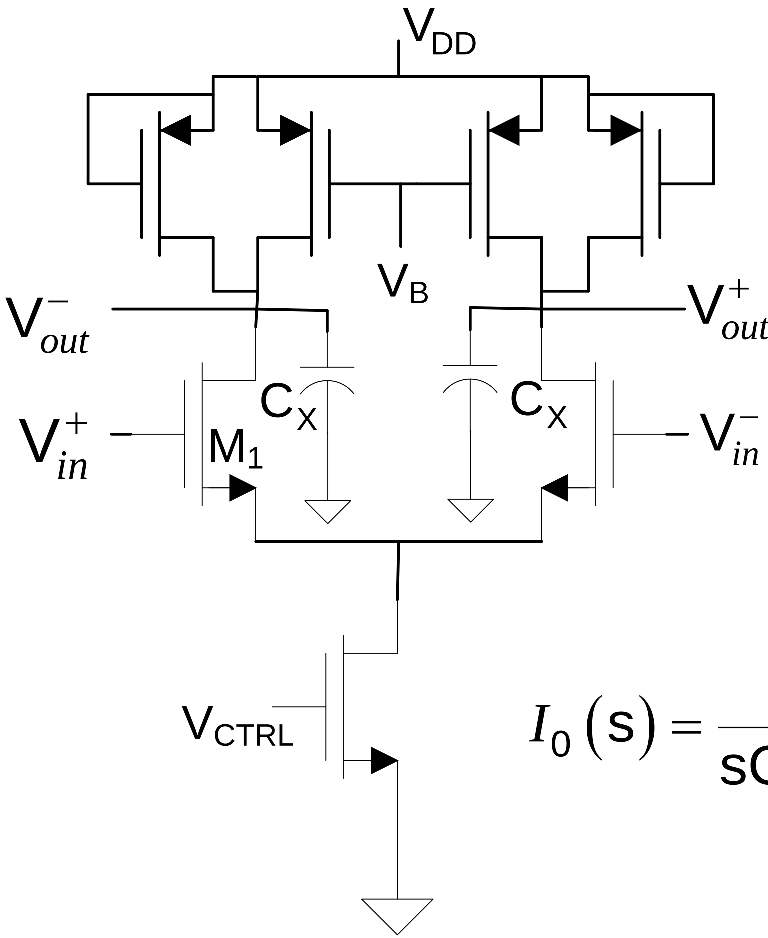
A fully-differential voltage-controlled integrator stage



$$I_{0d} = \frac{g_{m1}}{C_X}$$

Will need CMFB circuit

A fully-differential voltage-controlled integrator stage with loss



$$I_0(s) = \frac{g_{m1}}{sC_X + g_{m3}}$$

Will need CMFB circuit

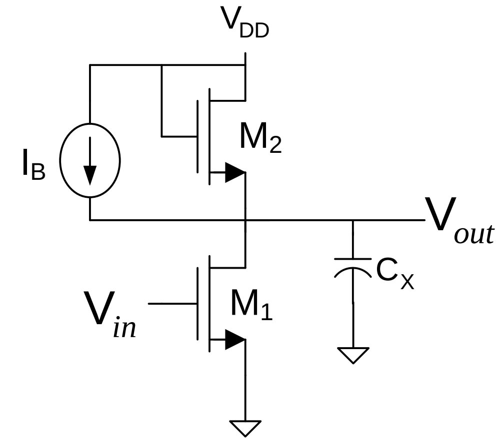
Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement

Recall:

$$I_0 = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (1)$$

$$\alpha_L = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \quad (2)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

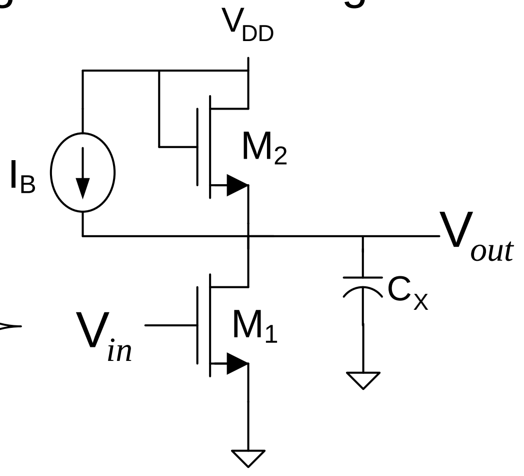
Substituting for I_0 and α_L we obtain:

$$\frac{g_{m1}}{C_X} = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (3)$$

$$\frac{g_{m2}}{C_X} = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \quad (4)$$

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



Expressing g_{m1} and g_{m2} in terms of design parameters:

$$\frac{\mu C_{OX} W_1 V_{EB1}}{L_1 C_X} = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (5)$$

$$\frac{\mu C_{OX} W_2 V_{EB2}}{L_2 C_X} = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \quad (6)$$

$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

If we assume I_B=0, equating drain currents obtain:

$$V_{EB2} = V_{EB1}\sqrt{\frac{W_1 L_2}{W_2 L_1}} \quad (7)$$

Thus the previous two expressions can be rewritten as :

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right] = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (8)$$

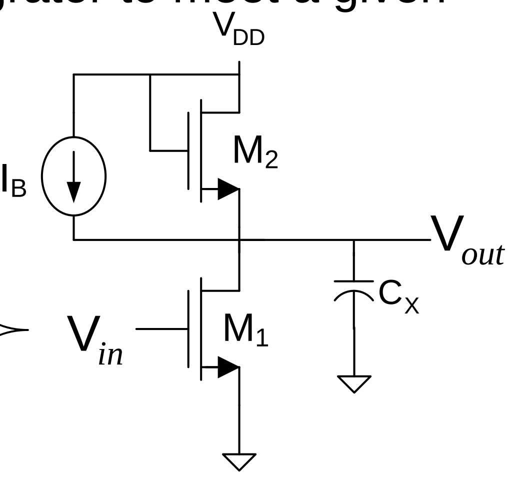
$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1 W_2}{L_1 L_2}\right] = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \quad (9)$$

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right] = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \qquad (8)$$

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1 W_2}{L_1 L_2}\right] = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \qquad (9)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

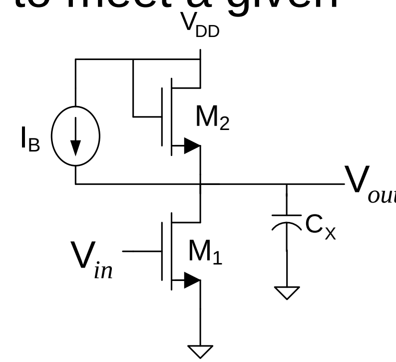
Taking the ratio of these two equations we obtain:

$$\frac{W_2}{L_2} = \frac{\sin\theta + \cos\theta\sqrt{4Q^2-1}}{\sqrt{4Q^2-1}} \qquad (10)$$

Observe that the pole Q is determined by the dimensions of the lossy device !

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right]=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \qquad (8)$$

$$\frac{W_2}{L_2}=\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}} \qquad (10)$$

Still must obtain W_1/L_1, V_{EB1}, and C_X from either of these equations

Although it appears that there might be 3 degrees of freedom left and only one constraint (one of these equations), if these integrators are connected in a loop, the operating point (Q-point) will be the same for all stages and will be that value where $V_{out}=V_{in}$. So, this adds a second constraint.

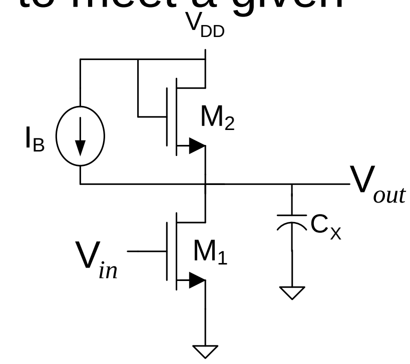
Setting $V_{out}=V_{in}$, and assuming $V_{T1}=V_{T2}$, we obtain from KVL

$$V_{DD}=V_{EB1}+V_{EB2}+2V_T \qquad (11)$$

But V_{EB1} and V_{EB2} are also related in (7)

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right]=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \qquad (8)$$

$$\frac{W_2}{L_2}=\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}} \qquad (10)$$

Still must obtain W_1/L_1, V_{EB1}, and C_X from either of these equations

$$V_{DD}=V_{EB1}+V_{EB2}+2V_T \qquad (11)$$

$$V_{EB2}=V_{EB1}\sqrt{\frac{W_1L_2}{W_2L_1}} \qquad (7)$$

$$\rightarrow \quad V_{EB1}=\frac{V_{DD}\text{-}2V_T}{1+\sqrt{\frac{W_2L_1}{W_1L_2}}} \qquad (12)$$

Substituting (10) into (12) and then into (8) we obtain

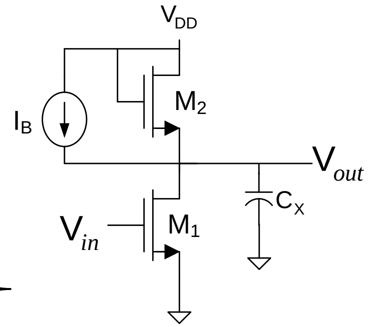
$$\frac{\mu C_{OX}}{C_X}\left[\frac{W_1}{L_1}\right]\left(\frac{V_{DD}-2V_T}{1+\sqrt{\left(\frac{W_1}{L_1}\right)^{-1}\left(\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}}\right)}}\right)=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \qquad (13)$$

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement

$$\frac{W_2}{L_2}=\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}} \quad (10)$$

$$\frac{\mu C_{OX}}{C_X}\left[\frac{W_1}{L_1}\right]\left(\frac{V_{DD}-2V_T}{1+\sqrt{\left(\frac{W_1}{L_1}\right)^{-1}\left(\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}}\right)}}\right)=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \quad (13)$$



There is still one degree of freedom remaining. Can either pick W_1/L_1 and solve for C_X or pick C_X and solve for W_1/L_1.

Explicit expression for W_1/L_1 not available

Tradeoffs between C_X and W_1/L_1 will often be made

Since $V_{OUTQ}=V_T+V_{EB1}$, it may be preferred to pick V_{EB1}, then solve (12) for W_1/L_1 and then solve (13) for C_X

Adding I_B will provide one additional degree of freedom and will relax the relationship between V_{OUTQ} and W_1/L_1 since (7) will be modified

Stay Safe and Stay Healthy !

End of Lecture 33